Online Appendix
Time-Varying Comovement of Foreign Exchange Markets

Mikio Ito[a], Akihiko Noda[b,c*] and Tatsuma Wada[c]

[a] *Faculty of Economics, Keio University, 2-15-45 Mita, Minato-ku, Tokyo 108-8345, Japan*

[b] *Faculty of Economics, Kyoto Sangyo University, Motoyama, Kamigamo, Kita-ku, Kyoto 603-8555, Japan*

[c] *Keio Economic Observatory, Keio University, 2-15-45 Mita, Minato-ku, Tokyo 108-8345, Japan*

[d] *Faculty of Policy Management, Keio University, 5322 Endo, Fujisawa, Kanagawa, 252-0882, Japan*

Abstract: This note provides detailed computations in the main text.

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[*] Corresponding Author. E-mail: noda@cc.kyoto-su.ac.jp, Tel. +81-75-705-1510, Fax. +81-75-705-3227.

A.1 Vector Error Correction Model as a Linear Regression Model

This technical appendix presents our econometric method to derive a time-varying speed of market integration for financial markets such as foreign exchange markets or commodity futures markets. It is based on a time-varying vector error correction (VEC) model and relies on an idea that a VEC model captures an adjustment process to a long run relationship. In particuar, a VEC model presents a change of cointegrated multivariate time series that is a sum of (1) the past stationary changes, (2) an error correction for the long run relashionship and (3) exhogenous shocks. Our aim is to estimate parameters involving with the error correction. Specifically speaking, we feature the loading matrix as time-varying parameters.

First, we present a framework of a VEC model to utilize simple least square techniques. Note first that a standard VEC model is derived from the following vector autoregression (VAR) equation for m-vector time series X_t $(t = 1, \cdots, T)$.

$$X_t = \Pi_1 X_{t-1} + \cdots + \Pi_k X_{t-k} + \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t, \tag{A.1}$$

where $\boldsymbol{\mu}$, D_t and $\boldsymbol{\varepsilon}_t$ denote a drift term, n exogenous vector (i.e. trend term) and a m random vector, respectively. Thus, Φ is $m \times n$ matrix; each Π_j's are $m \times m$ square matrices. Through tedeious algebra, we obtain the following VEC equation from provides

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \cdots + \Gamma_k \Delta X_{t-k+1} + \Pi_k X_{t-k} + \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t. \tag{A.2}$$

This equation suggests that the change of the time series, ΔX_t, is a sum of a stationary part, $\Gamma_1 \Delta X_{t-1} + \cdots + \Gamma_k \Delta X_{t-k+1} + \boldsymbol{\varepsilon}_t$, and an error correction $\Pi_k X_{t-k}$, which is stationary process when each component of X_t is an integrated process.

Supposing T sample periods, we can rewrite the equation (A.2) in an extended linear regression form. For example,

$$\begin{bmatrix} \Delta X_1 & \cdots & \Delta X_T \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mu} & \Gamma_1 & \cdots & \Gamma_k & \Pi \end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 \\ \Delta X_0 & \cdots & \Delta X_{T-1} \\ \vdots & \ddots & \vdots \\ \Delta X_{-k+1} & \cdots & \Delta X_{T-k+1} \\ X_{1-k} & \cdots & X_{T-k} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_1 & \cdots & \boldsymbol{\varepsilon}_T \end{bmatrix}.$$

Technically speaking, we can break down the above linear system to the followingthree cases according to what type of long run relations are spposed.

A.1.1 Error Correction Terms without Drift

The first case corresponds to the long run equilibrium equations without constant terms.

$$\begin{bmatrix}\Delta X_1 & \cdots & \Delta X_T\end{bmatrix} = \begin{bmatrix}\boldsymbol{\mu} & \Gamma_1 & \cdots & \Gamma_k\end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 \\ \Delta X_0 & \cdots & \Delta X_{T-1} \\ \vdots & \ddots & \vdots \\ \Delta X_{-k+1} & \cdots & \Delta X_{T-k+1} \end{bmatrix} + \Pi_{\mathrm{n}} \begin{bmatrix} X_{1-k} & \cdots & X_{T-k} \end{bmatrix} + \begin{bmatrix}\boldsymbol{\varepsilon}_1 & \cdots & \boldsymbol{\varepsilon}_T\end{bmatrix}. \quad \text{(A.3)}$$

A.1.2 Error Correction Terms with Drifts

The second case corresponds to the long run equilibrium equations with constant terms.

$$\begin{bmatrix}\Delta X_1 & \cdots & \Delta X_T\end{bmatrix} = \begin{bmatrix}\Gamma_1 & \cdots & \Gamma_k\end{bmatrix} \begin{bmatrix} \Delta X_0 & \cdots & \Delta X_{T-1} \\ \vdots & \ddots & \vdots \\ \Delta X_{-k+1} & \cdots & \Delta X_{T-k+1} \end{bmatrix} + \Pi_{\mathrm{c}} \begin{bmatrix} X_{1-k} & \cdots & X_{T-k} \\ 1 & \cdots & 1 \end{bmatrix} + \begin{bmatrix}\boldsymbol{\varepsilon}_1 & \cdots & \boldsymbol{\varepsilon}_T\end{bmatrix}. \quad \text{(A.4)}$$

A.1.3 Error Correction Terms with Linear Time Trend

The third case corresponds to the long run equilibrium equations with linear time trend.

$$\begin{bmatrix}\Delta X_1 & \cdots & \Delta X_T\end{bmatrix} = \begin{bmatrix}\boldsymbol{\mu} & \Gamma_1 & \cdots & \Gamma_k\end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 \\ \Delta X_0 & \cdots & \Delta X_{T-1} \\ \vdots & \ddots & \vdots \\ \Delta X_{-k+1} & \cdots & \Delta X_{T-k+1} \end{bmatrix} + \Pi_{\mathrm{t}} \begin{bmatrix} X_{1-k} & \cdots & X_{T-k} \\ 1 & \cdots & T \end{bmatrix} + \begin{bmatrix}\boldsymbol{\varepsilon}_1 & \cdots & \boldsymbol{\varepsilon}_T\end{bmatrix}, \quad \text{(A.5)}$$

where the last row of the second term in (A.5) presents a time trend $(1,\ 2,\ \cdots,\ T)$.

A.2 Least Square Technique for An Ordinary VEC Model

Notice that the dimension of the Π_{n} matrix for the third case differs from the other two cases. It is $m \times m$ whereas the others' are $m \times (m+1)$. Because a VEC model is algebraically derived from a certain VAR model, a linear stochastic system, it is also such a system. Thus, we can estimate parameters μ, Γ_i's and Π using some regresion techniques, say, OLS or GLS. Let Z_{0a}, Z_{1a} and $Z_{ka}, (a = \mathrm{n}, \mathrm{c}, \mathrm{t})$ denote appropriate data matrices representing for the equations (A.3), (A.4) and (A.5). Furether more, $\boldsymbol{e}$ denotes a matrix of exogenous shock vectors. That is,

$$Z_0' = \Gamma_{\mathrm{n}} Z_{1\mathrm{n}}' + \Pi_{\mathrm{n}} Z_{k\mathrm{n}}' + \boldsymbol{\varepsilon}, \tag{A.6}$$

where

$$Z_0' = \begin{bmatrix} \Delta X_1 & \cdots & \Delta X_T \end{bmatrix},$$

$$Z_{1\mathrm{n}}' = \begin{bmatrix} 1 & \cdots & 1 \\ \Delta X_0 & \cdots & \Delta X_{T-1} \\ \vdots & \ddots & \vdots \\ \Delta X_{-k+1} & \cdots & \Delta X_{T-k+1} \end{bmatrix},$$

$$Z_{k\mathrm{n}}' = \begin{bmatrix} X_{1-k} & \cdots & X_{T-k} \end{bmatrix},$$

$$\Gamma_{\mathrm{n}} = \begin{bmatrix} \boldsymbol{\mu} & \Gamma_1 & \cdots & \Gamma_k \end{bmatrix},$$

and

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \boldsymbol{\varepsilon}_1 & \cdots & \boldsymbol{\varepsilon}_T \end{bmatrix}.$$

Similarily, as for (A.4), its expression is as follows:

$$Z_0' = \Gamma_{\mathrm{c}} Z_{1\mathrm{c}}' + \Pi_{\mathrm{c}} Z_{k\mathrm{c}}' + \boldsymbol{\varepsilon}, \tag{A.7}$$

where

$$Z_{1\mathrm{c}}' = \begin{bmatrix} \Delta X_0 & \cdots & \Delta X_{T-1} \\ \vdots & \ddots & \vdots \\ \Delta X_{-k+1} & \cdots & \Delta X_{T-k+1} \end{bmatrix},$$

$$Z_{k\mathrm{c}}' = \begin{bmatrix} 1 & \cdots & 1 \\ X_{1-k} & \cdots & X_{T-k} \end{bmatrix},$$

and

$$\Gamma_{\mathrm{c}} = \begin{bmatrix} \Gamma_1 & \cdots & \Gamma_k \end{bmatrix}.$$

Finally, as for (A.4), its expression is as follows:

$$Z_0' = \Gamma_{\mathrm{t}} Z_{1\mathrm{t}}' + \Pi_{\mathrm{t}} Z_{k\mathrm{t}}' + \boldsymbol{\varepsilon}, \tag{A.8}$$

where

$$Z_{1\mathrm{t}}' = Z_{1\mathrm{n}}',$$

$$Z_{k\mathrm{t}}' = \begin{bmatrix} X_{1-k} & \cdots & X_{T-k} \\ 1 & \cdots & T \end{bmatrix},$$

and

$$\Gamma_{\mathrm{n}} = \Gamma_{\mathrm{t}}.$$

The three matrices, Γ_{n}, Γ_{c} and Γ_{t} might include $\boldsymbol{\mu}$ as well as $\Gamma_1, \cdots, \Gamma_k$. They provide us with information about stationary aspect of the time series X_t. On the other hand, the three matrices, Π_{n}, Π_{c} and Π_{t}, take a crucial role in a VEC model. They are decomposed into the *loading matrix* and the *cointegration matrix* such that $\Pi = \boldsymbol{\alpha\beta}'$. [1] In particular, $\boldsymbol{\beta}' Z_k'$ signifies some long run relationship among the observation. One can select the lag order k using usual information criteria such as SBIC for each linear models above; it is easy to compute them.

Given some cointegration order r, we can decompose the estimated Π in the above linear models with respect to Z_0, Z_1 and Z_K such that $\Pi = \boldsymbol{\alpha\beta}'$, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are called the loading and cointegration matrices, respectively. The cointegration order r is usually selected through well-known Johansen's test. Johansen's procedure on the rank helps ones to obtain all the estimates and statistics for applied econometrician.

A.3 Parameter Constancy Test and Random Walk Parameters

Since we can regard a VEC model as a simultaneous linear regression system, (A.6), (A.7) or (A.8), we can examine the possibility of parameter constancy on Π and Γ using Hansen's parameter constancy test (see Hansen (1992) for more details). Its null hypothesis is that parameters are time invariant; the alternative hypaothesis is that they are martingale.

There are several stochastic processes that are martingale. Thus, when the null hypothesis is rejected, we have to choose one of such processes that would be followed by the time-varying parameters in our model. Because we are interested in gradual changes in speed of adjustment to a certain long-run equilibrium for a VEC model, we choose a parameter dynamics in which the parameters follow random walk. The next subsection presents our estimation method for a VEC model with random walk prameters.

A.4 Time-Varying VEC Model

As Lütkepohl (2005) exactly points, the essential aim of VEC models is to decompose a multivariate time series into a pair of stationary and non-stationary time series. It is very similar to that of Beveridge and Nelson's (1981) decomposion for a univariate time series. Roughly speaking, we can consider that the matrices Γ_i's for $i = 1, \cdots, k$ represents the stationary structure of the time series to be analyzed; the matrix Π or matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ represent its non-stationary structure, the so-called cointegrated relationship among variables of the time series.

Although we start from a VAR model (A.1), its corresponding VEC model provides more information. When we consider a time-varying nature of some multivariate time

[1] Three identifier, "n", "c" and "t", for the above three options are omittied here.

series using VEC model, we should specify to what structure we focuss: that represented by Γ, that of Π or $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ or both.

This subsection presents some options for time-varying VEC model. Regarding a VEC model as a simple linear regression model, we just consider one of (A.6) through (A.8). Thus, we write down it for convenience as follows.

$$Z_0' = \Gamma Z_1' + \Pi Z_k' + \boldsymbol{\varepsilon}. \tag{A.9}$$

We can suppose following combinations of the parameter dynamics with the above linear model of VEC (A.9). First, we estimate Γ and Π as time-varying parameters without considering the decompositon of Π into $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

$$\Gamma_t = \Gamma_{t-1} + \boldsymbol{u}_t, \tag{A.10}$$

$$\Pi_t = \Pi_{t-1} + \boldsymbol{v}_t, \tag{A.11}$$

To this case, we can simply apply Ito et al.'s (2014) method to estimate a linear regression model.[2]

Second, we estimate Γ and $\boldsymbol{\alpha}$ with regarding $\boldsymbol{\beta}$ as time invariant and given.

$$\Gamma_t = \Gamma_{t-1} + \boldsymbol{u}_t, \tag{A.12}$$

$$\boldsymbol{\alpha}_t = \boldsymbol{\alpha}_{t-1} + \boldsymbol{v}_{1t}. \tag{A.13}$$

To this case, we first build a r dimensional time series $Y = Z_k\boldsymbol{\beta}$. Then, we apply Ito et al.'s method again to a new time linear regression

$$Z_0' = \Gamma Z_1' + \boldsymbol{\alpha} Y' + \boldsymbol{\varepsilon}, \tag{A.14}$$

considering (A.12) and (A.13).

Third, we estimate Γ and $\boldsymbol{\beta}$ with regarding $\boldsymbol{\alpha}$ as time invariant and given.

$$\Gamma_t = \Gamma_{t-1} + \boldsymbol{u}_t \tag{A.15}$$

$$\beta_t = \beta_{t-1} + \boldsymbol{v}_{2t} \tag{A.16}$$

Considering $\Pi = \boldsymbol{\alpha}\boldsymbol{\beta}'$, we first rewrite (A.9) as follows.

$$vec(Z_0) = (\boldsymbol{\alpha} \otimes Z_k)vec(\boldsymbol{\beta}) + (I \otimes Z_1)vec(\Gamma') + vec(\boldsymbol{\varepsilon}'), \tag{A.17}$$

where $\otimes$ is the Kronecker product and vec operator transforms a matrix into a vector by stacking the columns. Note that $\boldsymbol{\alpha} \otimes Z_k$ can be regarded as a data matrix becase $\boldsymbol{\alpha}$ is given. Considering this equation as a linear regression model whose parameters are supposed time-varying, we apply again the Ito et al.'s (2014) method estimate the parameters to be varying over time.

Note that both $\boldsymbol{\alpha}_t$ and $\boldsymbol{\beta}_t$ for each t cannot be estimated. In particular, since $\Pi_t = \boldsymbol{\alpha}_t\boldsymbol{\beta}_t'$ for each t and Π_t is not of full rank, a decomposion of Π_t into $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ is not unique. Thus, either $\boldsymbol{\alpha}_t$ or $\boldsymbol{\beta}_t$ is supposed time invariant for the most general case of both Π_t and Γ_t supposed time-varying.

[2] See their online appendix which is available at `http://at-noda.com/appendix/inter_market_appendix.pdf`.

A.5 Speed of Market Integration

We regard $\boldsymbol{\beta}' Z_k' = \mathbf{0}$ as long run equlibrium relations with respect to the multiple time series. The loading matrix $\boldsymbol{\alpha}$, representing a speed of adjustment, is time-varying when we use a time-varying VEC model to analyze market integration. Thus, we pay our attention on the time-varying loading maritrix $\boldsymbol{\alpha}_t$, which provides information about dynamics of market integration. The larger the absolute value of its components, the more significant their contribution to ameliorate deviation from the long run equilibrium. Thus, we propose an index of market integration based on the loading matrix. Then, we applied the index for the time-varying loading matrix $\boldsymbol{\alpha}_t$ to investigate how degree of market integration varies.

We derive the index ζ_t from $\boldsymbol{\alpha}_t$ following Ito et al. (2014). In particular,

$$\zeta_t = \sqrt{\max \lambda(\boldsymbol{\alpha}_t \boldsymbol{\alpha}_t')}. \tag{A.18}$$

That is, ζ_t is the square root of the largest eigen value of $\boldsymbol{\alpha}_t \boldsymbol{\alpha}_t'$, which is a non-negative semi definite matrix, for each t. Notice that the more the index the faster the adjustment of markets to the long run equilibrium.

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Table A.1: Asymptotic Critical Values for Hansen's (1992) Parameter Constancy Tests

DF(k)	Significance Level					
	1%	2.5%	5%	7.5%	10%	20%
1	0.74	0.58	0.46	0.39	0.35	0.24
2	1.07	0.89	0.75	0.67	0.61	0.47
3	1.36	1.16	1.00	0.91	0.84	0.68
4	1.62	1.41	1.24	1.14	1.06	0.88
5	1.87	1.64	1.46	1.36	1.28	1.08
6	2.12	1.88	1.69	1.57	1.49	1.27
7	2.35	2.10	1.90	1.78	1.69	1.46
8	2.58	2.32	2.11	1.99	1.89	1.65
9	2.81	2.54	2.33	2.19	2.10	1.84
10	3.04	2.76	2.53	2.40	2.29	2.03
11	3.26	2.97	2.74	2.60	2.49	2.22
12	3.47	3.18	2.94	2.79	2.69	2.40
13	3.69	3.39	3.14	2.99	2.88	2.59
14	3.90	3.59	3.34	3.19	3.07	2.77
15	4.12	3.80	3.54	3.39	3.27	2.96
16	4.33	4.00	3.74	3.58	3.46	3.14
17	4.54	4.21	3.94	3.77	3.65	3.32
18	4.74	4.41	4.13	3.96	3.84	3.50
19	4.95	4.61	4.33	4.16	4.03	3.69
20	5.16	4.81	4.52	4.35	4.22	3.87
21	5.36	5.01	4.72	4.54	4.41	4.05
22	5.57	5.21	4.91	4.73	4.59	4.23
23	5.77	5.40	5.11	4.92	4.78	4.41
24	5.98	5.60	5.30	5.11	4.97	4.59
25	6.17	5.80	5.49	5.30	5.15	4.77
26	6.38	5.99	5.68	5.49	5.34	4.95
27	6.58	6.19	5.87	5.67	5.52	5.12
28	6.77	6.38	6.06	5.86	5.71	5.30
29	6.98	6.58	6.25	6.05	5.90	5.48
30	7.18	6.77	6.44	6.23	6.08	5.66
31	7.38	6.97	6.63	6.42	6.26	5.84
32	7.56	7.15	6.82	6.61	6.45	6.02
33	7.76	7.35	7.01	6.79	6.63	6.20
34	7.97	7.55	7.19	6.98	6.81	6.37
35	8.17	7.74	7.38	7.16	7.00	6.55
36	8.36	7.93	7.57	7.35	7.18	6.73
37	8.55	8.12	7.76	7.53	7.36	6.90
38	8.74	8.31	7.94	7.71	7.54	7.08
39	8.94	8.50	8.13	7.90	7.72	7.26
40	9.14	8.69	8.32	8.08	7.91	7.44
41	9.34	8.88	8.50	8.27	8.09	7.61
42	9.52	9.06	8.69	8.45	8.27	7.79
43	9.72	9.25	8.87	8.63	8.45	7.96
44	9.91	9.44	9.06	8.82	8.63	8.14
45	10.11	9.64	9.25	9.00	8.81	8.32
46	10.29	9.82	9.43	9.18	8.99	8.49
47	10.48	10.01	9.61	9.37	9.18	8.67
48	10.68	10.20	9.80	9.55	9.36	8.85
49	10.87	10.39	9.98	9.73	9.54	9.02
50	11.06	10.57	10.17	9.91	9.72	9.20

Table A.1: (Continued)

DF(k)	Significance Level					
	1%	2.5%	5%	7.5%	10%	20%
51	11.25	10.76	10.35	10.09	9.90	9.37
52	11.45	10.95	10.54	10.27	10.08	9.55
53	11.63	11.13	10.72	10.45	10.25	9.72
54	11.83	11.32	10.90	10.64	10.44	9.90
55	12.01	11.51	11.09	10.82	10.62	10.07
56	12.21	11.70	11.27	11.00	10.80	10.25
57	12.40	11.88	11.45	11.18	10.98	10.42
58	12.59	12.07	11.64	11.36	11.16	10.60
59	12.78	12.25	11.82	11.54	11.33	10.77
60	12.96	12.43	12.00	11.72	11.51	10.95
61	13.16	12.63	12.18	11.90	11.69	11.12
62	13.34	12.81	12.37	12.08	11.87	11.30
63	13.53	13.00	12.55	12.27	12.05	11.47
64	13.72	13.18	12.73	12.45	12.23	11.65
65	13.91	13.36	12.91	12.62	12.41	11.82
66	14.09	13.55	13.10	12.81	12.59	12.00
67	14.29	13.74	13.28	12.99	12.76	12.17
68	14.47	13.92	13.46	13.16	12.94	12.35
69	14.65	14.10	13.64	13.34	13.12	12.52
70	14.85	14.29	13.82	13.52	13.30	12.69
71	15.04	14.47	14.00	13.71	13.48	12.87
72	15.22	14.65	14.18	13.88	13.66	13.04
73	15.41	14.84	14.36	14.06	13.83	13.21
74	15.59	15.02	14.54	14.24	14.01	13.39
75	15.77	15.20	14.72	14.42	14.19	13.56
76	15.97	15.39	14.91	14.60	14.37	13.74
77	16.15	15.57	15.09	14.78	14.54	13.91
78	16.34	15.75	15.27	14.96	14.72	14.08
79	16.52	15.93	15.44	15.13	14.90	14.26
80	16.71	16.12	15.63	15.31	15.08	14.43
81	16.90	16.31	15.81	15.49	15.26	14.60
82	17.08	16.49	15.99	15.67	15.43	14.78
83	17.27	16.67	16.17	15.85	15.61	14.95
84	17.46	16.85	16.35	16.03	15.79	15.13
85	17.64	17.03	16.53	16.21	15.96	15.30
86	17.82	17.22	16.71	16.39	16.14	15.48
87	18.01	17.40	16.89	16.56	16.32	15.65
88	18.20	17.58	17.07	16.75	16.50	15.82
89	18.39	17.77	17.25	16.92	16.67	15.99
90	18.57	17.95	17.43	17.10	16.85	16.17
91	18.75	18.13	17.61	17.27	17.02	16.34
92	18.93	18.31	17.79	17.45	17.20	16.51
93	19.12	18.50	17.97	17.63	17.38	16.69
94	19.30	18.67	18.14	17.81	17.55	16.86
95	19.48	18.85	18.33	17.99	17.73	17.03
96	19.67	19.04	18.50	18.17	17.91	17.21
97	19.86	19.22	18.69	18.34	18.08	17.38
98	20.04	19.40	18.86	18.52	18.26	17.55
99	20.22	19.58	19.04	18.70	18.44	17.73
100	20.41	19.76	19.22	18.87	18.61	17.90

Notes:

(1) Critical values were calculated from 20000 draws under the same setting as Hansen (1992).

(2) R version 3.3.1 was used to compute the critical values.

Table A.2: Asymptotic Critical Values for Qu's (2007) Cointegration Order Change Tests

q	α	$SupQ^1$	$SupQ^2$	WQ	SQ	$SupQ^F$	$SupQ^R$	$SupQ^W$
1	0.90	2.69	4.24	3.09	5.04	1.87	1.88	4.26
	0.95	3.15	4.98	3.60	5.81	2.45	2.45	5.17
	0.99	4.49	5.91	4.96	7.67	4.13	3.88	7.42
2	0.90	3.77	6.29	4.14	7.23	2.69	2.66	4.96
	0.95	4.39	7.12	4.73	8.30	3.27	3.14	5.69
	0.99	5.76	9.32	6.08	11.26	4.68	4.42	7.41
3	0.90	5.48	8.35	5.92	10.71	3.74	3.57	5.76
	0.95	6.06	9.12	6.34	11.60	4.31	4.08	6.38
	0.99	7.26	9.80	7.59	14.46	5.57	5.28	7.79
4	0.90	7.45	11.12	7.76	14.61	4.91	4.63	6.88
	0.95	8.13	11.88	8.36	15.78	5.46	5.16	7.50
	0.99	9.15	12.70	9.38	18.25	6.91	6.27	8.99
5	0.90	9.66	14.54	9.99	18.79	6.29	5.83	8.12
	0.95	10.23	15.03	10.59	19.80	6.94	6.37	8.71
	0.99	11.24	16.41	11.42	22.03	8.34	7.51	10.00
6	0.90	12.22	18.25	12.62	24.04	7.80	7.18	9.49
	0.95	12.83	19.04	13.21	25.35	8.53	7.71	10.04
	0.99	13.97	20.41	14.47	27.65	10.35	8.84	11.31
7	0.90	14.72	22.32	15.27	29.37	9.54	8.67	11.01
	0.95	15.26	23.57	15.75	30.23	10.35	9.25	11.58
	0.99	16.44	25.85	16.79	31.62	12.10	10.51	12.90
8	0.90	17.91	26.76	18.33	35.52	11.46	10.38	12.71
	0.95	18.53	27.55	19.05	36.61	12.38	11.00	13.26
	0.99	20.68	29.18	21.36	40.38	14.24	12.18	14.51
9	0.90	21.57	31.98	22.10	42.87	13.46	12.16	14.50
	0.95	22.35	32.82	22.86	44.19	14.35	12.79	15.08
	0.99	23.78	34.76	24.03	46.36	16.36	14.09	16.43
10	0.90	25.54	37.61	25.94	50.57	15.70	14.13	16.51
	0.95	26.33	38.21	26.91	52.10	16.69	14.74	17.11
	0.99	27.69	40.21	28.14	54.57	18.88	16.11	18.53
11	0.90	29.14	43.27	29.84	57.91	17.99	16.22	18.64
	0.95	30.26	44.49	30.68	59.67	19.08	16.88	19.26
	0.99	31.95	45.63	32.60	62.43	21.23	18.19	20.65
12	0.90	33.32	49.64	33.98	66.06	20.50	18.47	20.92
	0.95	34.14	50.45	34.49	67.66	21.65	19.14	21.62
	0.99	35.37	52.54	35.76	69.74	24.22	20.47	22.93
13	0.90	37.82	55.67	38.36	75.03	23.16	20.88	23.31
	0.95	38.61	56.47	39.29	76.80	24.47	21.62	24.05
	0.99	40.33	58.24	40.72	79.02	27.56	23.05	25.58
14	0.90	42.70	62.90	43.50	85.07	26.08	23.37	25.92
	0.95	43.96	64.30	44.62	86.87	27.51	24.16	26.58
	0.99	45.57	66.03	46.48	90.54	30.52	25.82	28.07
15	0.90	48.21	70.37	48.82	95.51	29.02	26.13	28.71
	0.95	48.93	71.33	49.42	97.31	30.62	26.93	29.44
	0.99	50.96	72.98	51.53	99.64	33.55	28.56	30.85
16	0.90	53.39	78.36	54.19	105.97	32.39	28.87	31.58
	0.95	54.50	79.74	55.28	108.46	33.89	29.67	32.27
	0.99	56.65	81.70	57.31	112.47	37.32	31.44	33.89
17	0.90	59.08	86.69	59.89	117.34	35.60	31.90	34.56
	0.95	60.15	87.87	60.78	119.19	37.15	32.78	35.31
	0.99	61.41	89.63	62.03	121.93	40.72	34.60	36.85
18	0.90	64.96	94.35	65.49	128.90	38.97	35.06	37.65
	0.95	65.69	95.11	66.44	130.52	40.60	35.95	38.44
	0.99	67.86	96.93	68.32	134.05	44.06	37.74	40.24
19	0.90	71.33	103.24	72.02	141.94	42.74	38.38	41.02
	0.95	72.27	104.05	73.00	143.89	44.58	39.28	41.89
	0.99	74.48	105.77	75.04	146.54	48.08	41.05	43.66
20	0.90	78.15	112.38	78.92	155.12	46.59	41.78	44.45
	0.95	79.27	113.71	79.88	156.93	48.31	42.78	45.32
	0.99	81.80	116.58	82.40	162.65	51.76	44.73	47.22

Notes:

(1) Critical values were calculated from 10000 draws under the same setting as Qu (2007).

(2) R version 3.3.1 was used to compute the critical values.